Power Allocation in Parallel Relay Channels using a Near-Potential Game Theoretical Approach

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***Abstract*—In this paper [1], the problem of distributed power allocation in a two-hop relay network is solved using a game theoretical approach. We consider a scenario, where each source node sends its packets to its corresponding destination via a preassigned relay node using half-duplex Amplify-and-Forward (AF) relaying method. Each source-relay-destination triplet is considered a player in the proposed game that attempts to maximize the end-to-end transmission rate of a source-destination link by allocating the available power to the source and its designated relay. A relay network with an arbitrary number of parallel links is considered. To maximize the link throughput, a near-potential game model for distributed power allocation over the communications link is proposed. Utilizing the near-potential game approach not only reduces the complex joint optimization problem to a single multi-variable function-maximization problem, but also enables us to prove the existence and uniqueness of the equilibrium result for the general case of N parallel links. The numerical analysis demonstrates the performance of this method that approaches the cooperative solution benchmark.**

I. Introduction

Noting the time-varying and random nature of wireless channels, a network user may adjust its parameters such as transmit power and coding rate in order to improve the communication efficiency. Competing benefits and limited resources of the users result in a competition among them. Hence, optimizing the overall system performance requires solving a joint optimization problem that finds the best resource allocation set for all users. The need for distributed resource allocation, in which the rational users optimize their own performance in the absence of a central controller, leads to adoption of game theoretical methodologies in wireless networks [1]–[4].

Recently, game theoretical approaches have been utilized to study resource allocation problems in relay interference channels. A non-cooperative power allocation framework for a relay network consisting of multiple source-destination pairs is proposed in [5], where all users share a single relay node. A pricing mechanism is designed to enforce the users to optimize their desired utilities in a distributed manner. This scenario does not cover the general case of multiple relay nodes. A more realistic two-user system ($N = 2$) is considered in [6], where a relay node is associated with each source-destination pair. In [6], the total power available to each link is fixed according to the assumption that each packet is allowed to consume a limited energy throughout its propagation form the source node to the destination node [7]. A distributed power allocation algorithm for Decode-and-Forward (DF) and Amplify-and-Forward (AF) relaying modes is proposed in [6], where the uniqueness of Nash Equilibrium (NE) is proved for special case of N=2 relay channels. However, this method is intractable for $N > 2$ and is not scalable to a system with an arbitrary large number of links.

A network of N source-relay-destination links is studied in [8], and an approximate best-response equilibrium for DF relaying mode is found. This result is approximate in the sense that the best-response result of user i at each round not only depends on the strategy of other users, but also is a function of its own strategy that is a non-causal requirement. Hence, this scheme is hinged on estimating the power and the algorithm leads to an approximate best-response result. Moreover, utilizing the proposed scalarization technique in [8], the uniqueness of the result can not be analytically proved and it is justified using numerical methods. Using the standard game theoretical approach, a strict sufficient condition on the system parameters is derived to guarantee the uniqueness of the result. This condition requires that the normalized interference channel coefficients of one hop completely dominate the channel gains of the other hop, such as the case that the relay nodes are either very close to the source or destination. This condition limits the application of this method in realistic systems. An implicit-based algorithm to prove the uniqueness of the NE equilibrium for a N-user two-hop interference channel deploying DF relaying method at the relays is proposed in [9]. In [10], a distributed power allocation game model is proposed to maximize the users' throughput considering the fairness in resource allocation.

The problem of power allocation in a network with multiple AF cooperative links is studied in [11], where a two-stage game model is proposed to optimize the transmission power of the relay nodes. In this model, the users form coalitions in the first stage and engage in a non-cooperative game in the second stage noting the output coalitions structure of the first stage. In this scenario, the users are allowed to coordinate with each other to manage a proper time division multiple access scheme and reduce the interference, in the sense that only one link will be active within each coalition. In other words, this model requires a coordination between the users to decide which user can be active in each coalition that imposes a heavy signaling to the system. This is even more critical for a large number of users to perform the arrangement.

In this paper, power allocation in an N-user parallel relay interference network deploying AF relaying mode is studied, where no coordination communication is allowed between the links. A near-potential game model is proposed to obtain the optimal power allocation set of the source and relay nodes in high Signal to Interference and Noise (SINR) regime. Using this approach not only guarantees the uniqueness of the equilibrium solution for an arbitrary number of users, but also

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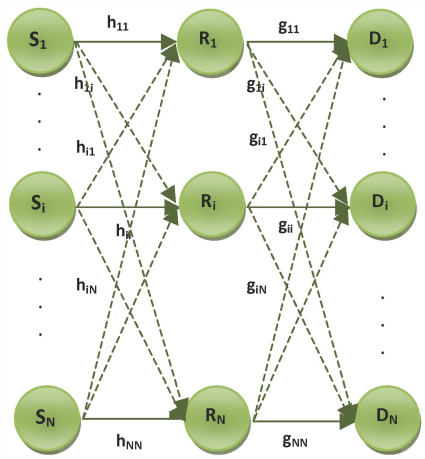


Fig. 1: System model for relay interference channel composed of N source-relay-destination (SRD) links.

simplifies the complex problem of jointly optimizing N utility functions to find the local maximum of a single near-potential function. The numerical results demonstrate the closeness of the proposed game results to those of best response method and the cooperative solution as a benchmark solution.

II. System Model

The system model consists of N parallel relay links as depicted in Fig. 1. The link i, includes a source node, S_i, its corresponding destination node, D_i and a preselected relay node, R_i. AF relaying is performed at the relay nodes. It is assumed that the source nodes do not directly communicate with destinations, due to their limited communication range or shadowing effects [12]–[14]. Each packet transmission takes place at two phases. At the first time slot, source nodes transmit their packets to the corresponding relay nodes over the *first hop* of the communication link. At the subsequent time slot, the relay nodes amplify the previously received signals and forward them to the destinations over the *second hop* of the communication link; hence there is no interference between these two stages. However, since all source-to-relay channels share the same spectrum, this causes interference among source-relay communication links. The same statement applies to the relay-destination channels, where interference is considered as well.

A slow fading scenario is considered, meaning that the channel coefficients are fixed during one transmission phase. Also, at both transmission stages, the received interference is treated as noise [6], [8], that is considered a practical method in most applications due to the complexity of channel estimation for interference cancellation [15]. A limited energy is assumed for each packet propagation from its source to the destination as in [6], [7], [12]. Noting the fixed frame length and half duplex relaying method, this assumption is equivalent to a fixed power constraint on the transmission power of each source and relay pair as [6], [12]

$$P_i^{(S)} + P_i^{(R)} \leq P_i \tag{1}$$

where $P_i^{(S)}$, $P_i^{(R)}$ and P_i denote the transmit power of the ith source and relay nodes, and the available power of link i, respectively.

The transmit symbol of source i is denoted by x_i. A unit average-energy constellation is assumed ($E[|x_i|^2] = 1$). The received message by relay node i is:

$$y_i^{(R)} = h_{ii}\sqrt{P_i^{(S)}}x_i + \sum_{j=1, j\neq i}^{N} h_{ji}\sqrt{P_j^{(S)}}x_j + n_i^{(R)} \tag{2}$$

where h_{ij} represents the channel coefficient between the source i and the relay j and $n_i^{(R)}$ is the equivalent Additive White Gaussian Noise (AWGN) term with variance σ_{i1}^2 at the relay i. The received signal at destination i is

$$y_i^{(D)} = A_i g_{ii} y_i^{(R)} + \sum_{j=1, j\neq i}^{N} A_j g_{ji} y_j^{(R)} + n_i^{(D)} \tag{3}$$

where g_{ij} and $n_i^{(D)}$ denote the channel coefficients from relay i to destination j and the AWGN noise term with variance σ_{i2}^2 at the destination, respectively. A_i is the amplifier gain of relay i, which is

$$A_i = \sqrt{\frac{P_i^{(R)}}{|y_i^{(R)}|}} = \sqrt{\frac{P_i^{(R)}}{\sum_{k=1}^{N}(P_k^{(S)}|h_{ki}|^2) + \sigma_{i1}^2}} \tag{4}$$

The maximum achievable rate of a two-hop link using AF relaying according to [16] is calculated as

$$I_i(\mathbf{p}_i) = \frac{1}{2}\log_2\big(1 + \frac{\gamma_i^{(SR)}\gamma_i^{(RD)}}{\gamma_i^{(SR)} + \gamma_i^{(RD)} + 1}\big), \tag{5}$$

where $\mathbf{p}_i = [P_i^{(S)}, P_i^{(R)}]$, and $\gamma_i^{(SR)}$ and $\gamma_i^{(RD)}$ denote the SINRs of the first and second hops of link i, as defined by the following equations:

$$\gamma_i^{(SR)} = \frac{|h_{ii}|^2 P_i^{(S)}}{\sum_{\substack{j=1\\ j\neq i}}^{N}(|h_{ji}|^2 P_j^{(S)}) + \sigma_{i1}^2}, \tag{6}$$

$$\gamma_i^{(RD)} = \frac{|g_{ii}|^2 P_i^{(R)}}{\sum_{\substack{j=1\\ j\neq i}}^{N}(|g_{ji}|^2 P_j^{(R)}) + \sigma_{i2}^2}. \tag{7}$$

The transmission rate for link i depends on how the available power P_i is allotted to the two communication hops. The optimal power allocation for the non-cooperative scenario is found through a near-potential game theoretical approach as discussed in section III.

III. Near-Potential Game Model

In the potential game approach, any change in the utility function of a player due to unilateral change of its strategy is represented in a global potential function. Consequently, the existence of a pure Nash equilibrium is guaranteed, provided that the potential function necessary requirements are met. Moreover, in these games, most dynamic processes, including best-response and better-response, converge to a unique NE [17], [18]. Potential games have been recently considered in the literature to model different problems in communication networks [19]. A distributed-resource allocation algorithm based on potential game is proposed in [20], where the efficiency and stability of Nash equilibrium are studied. A framework based on potential game for resource allocation problem is introduced in [21], which covers a more general case of games with coupled strategies.

Due to the restricting conditions of potential games, most of the optimization problems in communication networks can not be modeled by these games. However, noting the considerable benefits of potential games a new approach of near-potential games has been introduced in [22] and [23]. These games are derivations of potential games, with appropriate convergence properties, but milder game conditions, that cover a wider category of applications. The convergence of different learning mechanisms for near-potential games is studied in [24]. In this scenario, the near-potential game solution approaches to a pure approximate equilibrium called $\epsilon-$equilibrium, that is approximately a Nash equilibrium. According to [25], a strategy profile, s^* achieves $\epsilon-$equilibrium if,

$$\forall i \in Q_N,\ \forall s_i \in S_i:\ u_i(s_i^*,\ s_{-i}^*) \geq u_i(s_i,\ s_{-i}^*) - \epsilon \tag{8}$$

where Q_N is the set of players, s_i is the strategy of player i and S_i denotes the strategy set of player i.

A distributed near-potential game model is proposed in this section to solve the power allocation problem for relay interference channels in a high SINR regime. The non-cooperative game model for power allocation in relay interference channel does not meet the required conditions to be a potential game; however the symmetric property and the format of transmission rate specially in high SINR motivated us to propose a near-potential game approach, which reflects the changes in the original utility functions for different strategies with an acceptable approximation. The problem of non-cooperative power optimization for N-parallel relay links, where each link only considers its individual welfare to maximize its transmission rate can be formulated as

$$\begin{aligned} &\max_{\mathbf{p}_i}\{I_i(\mathbf{p}_1,\cdots,\mathbf{p}_N)\} = \max_{\mathbf{p}_i}\{I_i(\mathbf{p}_i)\} \\ &\text{subject to} \begin{cases} 0 < P_i^{(S)}, P_i^{(R)} \leq P_i, \\ P_i^{(S)} + P_i^{(R)} \leq P_i, \qquad \forall i \in Q_N \end{cases} \end{aligned} \tag{9}$$

This optimization problem can be modeled by game, $G =< Q_N, \{\mathbf{p}_i\}, \{u_i\} >$, $i \in Q_N$, where

- $Q_N = \{1, 2, ..., N\}$ is the finite set of players.
- $\mathbf{p}_i$ is the strategy set of player i. Notation $\mathbf{p}_{-i}$ denotes the strategy sets of all players except player i.
- u_i is the utility function of player i.

Each two-hop link between the source and destination is considered a game player. Pure strategy of player i is defined as the power vector of the ith link, denoted by $\mathbf{p}_i = (P_i^{(S)}, P_i^{(R)})$. Consequently, the strategy profile of the game is $\mathbf{p} = \mathbf{p_1} \times \mathbf{p_2} \times ... \times \mathbf{p_N}$, which determines the power of all source and relay nodes in the system. It is evident from (6) that in non-cooperative scenario, regardless of the power allocation of other links, the maximum rate for link i is achieved if the whole available power is exhausted at this link. Therefore, considering the fixed power for each link P_i in (1), we have $P_i^{(S)} + P_i^{(R)} = P_i$ and the power allocation of link i can be fully identified by a single power ratio parameter defined as

$$\alpha_i = \frac{P_i^{(S)}}{P_i^{(S)} + P_i^{(R)}} \tag{10}$$

In high SINR regime, the noise term is negligible and $|h_{ii}| \gg |h_{ij}|, |g_{ii}| \gg |g_{ij}|$ for $\forall i \neq j$. Hence, $\gamma_i^{(SR)} \gg 1$, $\gamma_i^{(RD)} \gg 1$, $\gamma_i^{(SR)} + \gamma_i^{(RD)} + 1 \cong \gamma_i^{(SR)} + \gamma_i^{(RD)}$. So noting $X \gg 1 \Rightarrow \log(1+X) \cong \log(X)$, the transmission rate of link i in (5) can be approximated by

$$I_i(\mathbf{p}_i, \mathbf{p}_{-i}) = -\frac{1}{2}\log_2\left[\frac{1}{\gamma_i^{(SR)}} + \frac{1}{\gamma_i^{(RD)}}\right] \tag{11}$$

In the proposed game model, the utility function of player i is defined as the approximate transmission rate as presented in (11) and the game G for each player is formulated as follows:

$$G: \begin{cases} \max_{p_i}[\ u_i(\mathbf{p}_i, \mathbf{p}_{-i})\ ], \\ s.t.\ 0 \leq \mathbf{p}_i = (P_i^{(S)}, P_i^{(R)}) \leq P_i, \forall i \in Q_N. \end{cases} \tag{12}$$

Proposition 1: Following the necessary and sufficient condition of a game with twice differentiable utility functions to be a potential type, presented in [17], the game model, G can not be a potential one, since

$$\frac{\partial^2 u_i}{\partial P_i^{(S)} \partial P_j^{(S)}} \neq \frac{\partial^2 u_j}{\partial P_j^{(S)} \partial P_i^{(S)}}, \quad \forall i, j \in Q_N. \tag{13}$$

However, the logarithmic format of the utility function and also its symmetric property due to same distribution of the channel coefficients of two stages, h_{ij} and g_{ij}, $\forall i, j \in Q_N$, lead us to propose the following near-potential function for power allocation.

$$\Phi(\mathbf{p}) = \frac{1}{\sum_{i=1}^{N}\left(\frac{1}{|h_{ii}|^2 P_i^{(S)}} + \frac{1}{|g_{ii}|^2 P_i^{(R)}}\right)} \tag{14}$$

Utilizing the near-potential game approach the existence and uniqueness of the equilibrium point is proven as follows.

Theorem 1: The proposed near-potential function is strictly concave on $P_i^{(S)} \in [0, P_i]$.

Proof: See Appendix A. ■

Theorem 2: The proposed near-potential game has a unique equilibrium.

Proof: See Appendix B. ■

Applying the best-response dynamic to the proposed game model makes the solution converge to the unique maximum of the near-potential function, which is in a neighborhood of the original game NE. In best-response dynamics, players are myopic in the sense that in each round of playing the game, they consider only the current benefits and not the ones corresponding to the previous rounds nor the future of the game. Also, in each round, only one player updates its strategy to maximize its utility given the strategies of its opponents are fixed. In this work, we have used the Round Robin fashion and sequentially chose the users in repeated cycles. In best-response dynamic, in round k, player i updates its strategy s_i^k as

$$s_i^k = \arg\max_{s_i' \in \mathbf{S}_i} u_i(s_i', \mathbf{s}_{-i}^{k-1}) \tag{15}$$

Applying the near-potential game approach, the best-response of any player can be found by maximizing the corresponding near-potential function given the strategies of its opponents as follows

$$s_i^k = \arg\max_{s_i' \in \mathbf{S}_i} \phi(s_i', \mathbf{s}_{-i}^{k-1}) \tag{16}$$

Since the proposed near-potential function is twice continuously differentiable and strictly concave on a convex strategy set of the game strategy set, $\mathbf{p}$, hence the set of maximizers of the near-potential function Φ coincides with the set of ϵ-equilibriums of the game, and the best response dynamic converges to the maximum of the near-potential function, which is a unique pure approximate equilibrium in a close neighborhood of the original game's NE.

IV. Numerical Results

In this section, the numerical results are presented to validate the performance of the proposed game solution. In all simulations, end-to-end transmission powers are normalized to $P_1 = P_2 = ... = P_N = P = 20$. However, to account for unequal power-to-noise ratios in heterogeneous systems, different noise variances are assumed for each hop. Therefore, n_{il} is additive zero mean Gaussian noise with variance σ_{il}^2 for $i \in Q_N,\ l \in \{1, 2\}$. The parameter σ_{il}^2 is a random variable drawn from an Inverse-Gamma distribution with shape and scale parameters $a = b = 1$ as a commonly used model for Gaussian noise variance [26]. Channel coefficients h_{ij} and g_{ij} are drawn from a Rayleigh distribution with scale parameter 1. The non-diagonal elements of the channel matrix are attenuated by a factor of 10 to ensure moderate-to-high SINR regime. Therefore, we have $E[\frac{h_{ii}^2 P_i/2}{\sigma^2}] = 10$ and $E[\frac{h_{ii}^2}{h_{ij}^2}] = E[\frac{g_{ii}^2}{g_{ij}^2}] = 10$ for $i \neq j$. Finally, we have set the maximum number of iterations to 10, since simulations demonstrate that the game reaches its final solution after a few iterations.

Fig. 2 shows a typical shape of the near-potential function as derived in (14) for a two user system ($N = 2$). It is evident from this figure that the near-potential function is a strictly concave function of power allocation parameters, α_1, α_2. This property guarantees the existence and uniqueness of the equilibrium point. Indeed, in this approach, the nodes intend to follow the global maximum of the near-potential function. It is shown next that this solution is very close to the best-response NE and cooperative solutions in terms of system achievable rate.

In Fig. 3, the near-potential game and the best-response solutions are provided for a six-user system. In this figure, the results are corresponding to the last iteration of the game, where the system is settled at the final solution. The solid *blue* line is the utility function of each link to show the the best-response (BR) approach. The utility function is the end-to-end transmission rate, assuming a fixed power allocation for other links. Therefore, the maximum of this curve is corresponding to the best-response NE power allocation. This solution is depicted by dashed *blue* line in this figure.

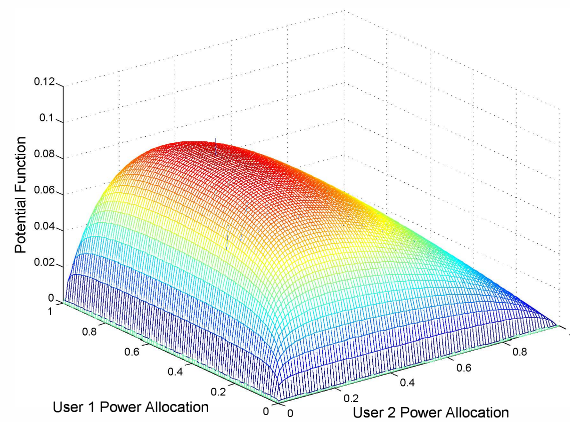
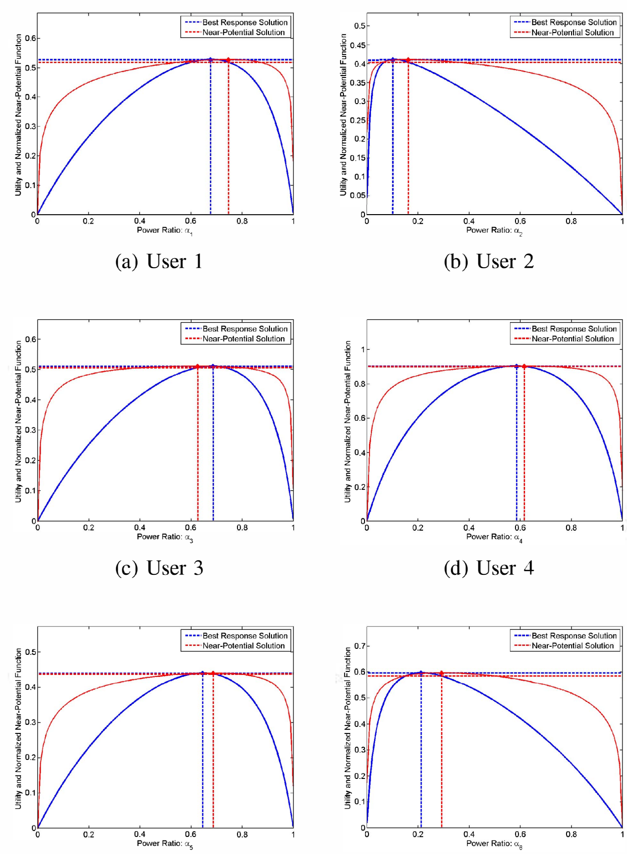


Fig. 2: Proposed near-potential function versus the power allocation parameters, α_1 and α_2, for a two-user relay interference network.



(a) User 1 (b) User 2

(c) User 3 (d) User 4

(e) User 5 (f) User 6

Fig. 3: Utility function and near-potential function solutions to find optimal power allocation for a six-user relay interference network.

TABLE I: Comparison between solutions and corresponding rates of the proposed near-potential (NP) game method with best-response (BR) and cooperative (CO) methods in a six-user network.

Solution Type	Cooperative (CO)	Best-response (BR)	Near-Potential (NP)
$\alpha = \alpha_1$	0.68191	0.67641	0.74698
α_2	0.10113	0.10181	0.16229
α_3	0.67523	0.68649	0.62601
α_4	0.58511	0.58569	0.61593
α_5	0.64853	0.64617	0.68649
α_6	0.21128	0.2127	0.29334
$\mathbf{u} = u_1$	0.54406	0.52805	0.51851
u_2	0.42388	0.41132	0.40607
u_3	0.52683	0.51159	0.50634
u_4	0.93108	0.904	0.90057
u_5	0.45427	0.44104	0.43868
u_6	0.61521	0.59725	0.58672
$\mathbf{u_{sum}} = \sum u_i$	3.4953	3.3933	3.3569
dev($\mathbf{BR/NP}$)	-	2.92%	3.96%

Similarly, the solid *red* line is the normalized near-potential function versus the relevant user's power allocation parameter given the other users' power allocation parameters fixed. The near-potential function is normalized for the sake of comparison simplicity as $\phi(\alpha_i) = \frac{max\{u_i(\alpha_i)\}}{max\{\phi(\alpha_i)\}}\phi(\alpha_i)$. The maximum point of this curve defines the power allocation ratio obtained using the potential game approach. It is noteworthy that once the optimal power ratio is found by maximizing the potential function, the actual rate of the link is obtained by evaluating the rate function (blue curve) at this point. In simple words, the difference between the vertical dashed lines are the difference between power allocation parameters for the best response and near-potential approach, while the difference between the two horizontal dashed lines is the degradation of the link rate in near-potential approach with respect to the best response method. The results clearly demonstrate that even though the near-potential function does not exactly follow the utility function (since it is not an exact potential function), the resulting power allocation is very close to the best-response NE solution at the last iteration. In fact, for user 3, 4 and 5, almost the same power allocation is obtained from both solutions. For the rest of users, the power allocation parameter obtained from the two approaches are slightly different. However, the final outcomes of the proposed game, which are the corresponding end-to-end transmission rates are almost the same.

The obtained results of the non-cooperative solutions including both best response and near-potential game are compared to the optimal cooperative solution in Table I. The first row shows the optimum power ratio for six users obtained from cooperative (CO), best-response (BR) and near-potential (NP) approaches. The utility of each user is calculated as the link end-to-end transmission rate for each of the above approaches, presented in the second row. These results clearly show that the power allocation of each user found by using the near-potential game method yields similar utility with the one found from the best-response solution. Consequently, the resulting total sum rates for both solutions are shown to be very close as well.

For a clear comparison, the deviation of the NP and BR approaches from the optimal solution (CO) is defined as dev$(BR/NP) = |\frac{\sum_{i=1}^{n} u_i(\alpha_i^{BR/NP}) - \sum_{i=1}^{n} u_i(\alpha_i^{CO})}{\sum_{i=1}^{n} u_i(\alpha_i^{CO})}|$. The results in Table I demonstrate that even though the transmission rate of each user obtained from the near-potential and best-response solutions might be different than those of cooperative solution, the achieved sum rates are very close and show negligible degradation from the optimal solution $\big(dev(BR), dev(NP) < 4\%\big)$. More specifically, the average degradation of the sum rate by replacing the classical best response approach with the proposed Near potential solution is about 1%. It is noteworthy, that CO solution suffers from two major issues of i) the need for a central controller and ii) computational complexity. If the power range is divided into L bins and the number of users is N, the CO method involves L^N search steps, which is not practically feasible for relatively large number of users.

V. Conclusions

In this paper, a network of N parallel source-relay-destination links utilizing AF relaying method at the relay nodes is considered. A near-potential game model is proposed to find the optimal power allocation solution in this network. This method simplifies the joint power optimization problem to a less complex one of maximizing a single multi-variable near-potential function. The proposed near-potential function reflects the changes in the original utility functions for different strategies with an acceptable approximation. Utilizing the near-potential game approach the proof of existence and uniqueness of the solution is presented for an arbitrary number of users. Despite considerable reduction in the complexity of algorithm, no degradation in the data rates are observed with respect to the best-response solution. The obtained results are almost equal to the more complex best response solution and are in the 5% vicinity of the benchmark cooperative solution.

Appendix *A*

Proof: In this part, the proof of concavity for the near-potential function is provided. Let f be a function of many variables defined on the convex set S. Then the function f is concave on the set S if and only if for $\forall x \in S$, $\forall \bar{x} \in S$, and $\forall \lambda \in (0,1)$ we have [27],

$$f((1-\lambda)x + \lambda\bar{x}) \geq (1-\lambda)f(x) + \lambda f(\bar{x}) \quad (17)$$

If $\forall x \neq \bar{x}$, the definition of concavity in (17) is held with a strict inequality ($>$ rather than $\geq$), then f is strictly concave on S.

In seeking the maximum of a potential function, in each round of power optimization, only the power of one user will be changed and the other users' powers remain fixed. Hence, if the potential function is strictly concave in each user's unilateral deviation, the corresponding equilibrium is unique. Consequently, the proof is completed by verifying the strict concavity of the potential function with respect to any arbitrary power parameter given the others are fixed. By symmetry of the near-potential function proposed in (14), we examine for an arbitrary user i power as:

$$\Phi(p_i, \mathbf{p_i}) = \frac{1}{\frac{1}{|h_{ii}|^2 P_i^{(S)}} + \frac{1}{|g_{ii}|^2 (P_i - P_i^{(S)})} + K} \quad (18)$$

where $K = \sum_{j=1, j\neq i}^{N} (\frac{1}{|h_{jj}|^2 P_j^{(S)}} + \frac{1}{|g_{jj}|^2 P_j^{(R)}})$ is a constant.

Since (18) is twice differentiable on $[0, P_i]$, therefore according to [27], $\Phi(p_i, \mathbf{p_i})$ is strictly concave if and only if $\frac{\partial^2 \Phi}{\partial {P_i^{(S)}}^2} < 0$. We proceed to check this condition for the proposed near-potential function,

$$\frac{\partial \Phi}{\partial P_i^{(S)}} = -\frac{u}{v^2} \tag{19}$$

where

$$u = -\frac{1}{|h_{ii}|^2 {P_i^{(S)}}^2} + \frac{1}{|g_{ii}|^2 (P_i - P_i^{(S)})^2}, \tag{20}$$

$$v = \frac{1}{|h_{ii}|^2 P_i^{(S)}} + \frac{1}{|g_{ii}|^2 (P_i - P_i^{(S)}) + K} \tag{21}$$

Taking second derivative yields

$$\begin{aligned}
\frac{\partial^2 \Phi}{\partial {P_i^{(S)}}^2} &= -\frac{\frac{\partial u}{\partial P_i^{(S)}} v^2 - 2uv \frac{\partial v}{\partial P_i^{(S)}}}{v^4} \\
&= -\frac{2}{\left(\frac{1}{|h_{ii}|^2 P_i^{(S)}} + \frac{1}{|g_{ii}|^2 (P_i - P_i^{(S)})} + K\right)^3} \\
&\times \Big(\frac{1}{|h_{ii} g_{ii}|^2 (P_i^{(S)})^3 (P_i - P_i^{(S)})} \cdots \\
&+ \frac{1}{|h_{ii} g_{ii}|^2 P_i^{(S)} (P_i - P_i^{(S)})^3} \cdots \\
&+ \frac{2}{|h_{ii} g_{ii}|^2 (P_i^{(S)})^2 (P_i - P_i^{(S)})^2} \cdots \\
&+ \frac{K}{|h_{ii}|^2 (P_i^{(S)})^3} + \frac{K}{|g_{ii}|^2 (P_i - P_i^{(S)})^3} \Big)
\end{aligned} \tag{22}$$

All negative terms cancel out and both numerator and denominator only include positive terms. Therefore, $\frac{\partial^2 \Phi}{\partial {P_i^{(S)}}^2} < 0$, which means the proposed near-potential function is strictly concave. ■

Appendix *B*

Proof: Here we justify that the proposed near-potential game model has a unique equilibrium. Since the strategy set of the proposed game, $\mathbf{p} = \mathbf{p_1} \times \mathbf{p_2} \times ... \times \mathbf{p_N}$ is convex, and also the proposed near-potential function in (14) is twice continuously differentiable and concave on $P_i^{(S)}$, the set of maximizers of the near-potential function Φ coincides with the set of ϵ-equilibriums of the game. In addition, since the near-potential function Φ is strictly concave on S, the potential function has a unique maximum and consequently, the equilibrium of the game is unique. ■

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