Binary Source Estimation Using a Two-Tiered Wireless Sensor Network

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***Abstract*—Binary source estimation using indirect observation by a cluster of wireless sensors in a two-tiered architecture is considered. The high correlation among sensors' observation is exploited in order to implement an efficient distributed Joint Source Channel Coding (DJSCC) scheme based on Parallel Concatenated Convolutional Codes (PCCC). A novel approach employing two sink nodes per cluster providing a Distributed Space-Time Block Coding (DSTBC) is proposed to get diversity gain over fading channels without need to set up an extra link between two sinks. The proposed iterative decoding algorithm is easily scalable to a large number of sensors and outperforms independent RSC decoders followed by a Maximum Likelihood (ML) detector.**

***Index Terms*—Two-tired wireless sensor networks, distributed codes, binary source estimation, iterative decoding.**

I. INTRODUCTION

IN a clustered Wireless Sensor Network (WSN) sensing a continuous variation data field, sensors' observations are highly correlated. One commonly used model for this correlation is Binary Symmetric Channels (BSC) [1], which is considered in the proposed system model. The correlation among sensors' data has been widely used to improve source coding efficiency according to Slepian-Wolf theorem [2]. In addition, combining source coding with channel coding can simplify the system [3].

One special case is to estimate a single data source using different observations, which is called Chief Executive Officer (CEO) problem [4]. The number of required sensors is imposed by the observation accuracy and the desired BER performance. This number can be fairly high for applications where the sensors' observations are not accurate [5]. Most of the reported DJSCC codes in the literature are based on LDPC and Turbo codes that are not easily scalable to a large number of sensors due to their decoding complexity [6].

In this paper, a novel DJSCC scheme for CEO problem is proposed which has low complexity and is highly scalable. The system composed of simple structured sensors equipped with random interleavers and RSC encoders. The proposed iterative decoding algorithm estimates the observation accuracy parameter and uses it in decoding process. To further improve the system performance, two sink nodes operating at half power mode are located in each cluster and DSTBC is employed to increase the system performance over Rayleigh fading channels.

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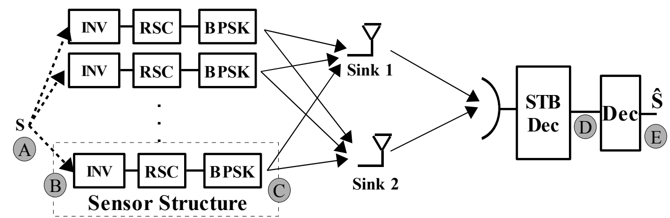

Fig. 1. System model: a binary source observed by a tow-tiered wireless sensor network.

II. DISTRIBUTED CODING OF BINARY SOURCES

A. Correlation Model

A cluster of M sensors that are observing the same source is considered. The binary source data frame $S = \{s^1, s^2, ..., s^N\}$ is assumed to be an independent identically distributed $(i.i.d)$ Bernoulli sequence of length N with $p(s^k = 0) = p(s^k = 1) = 1/2$. Sensor i measures a noisy version of data denoted by $X_i = \{x_i^1, x_i^2, ..., x_i^N\}$. The channel error between source data S and data observed by the i^{th} sensor is modeled with a BSC channel with crossover probability β_i; therefore the k^{th} bit in X_i is

$$x_i^k = s^k \oplus e_i^k \tag{1}$$

where e_i^k is the error in bit k of the i^{th} sensor with probability of $p(e_i^k = 1) = \beta_i,\ p(e_i^k = 0) = 1 - \beta_i$, for $i = 1, 2, ..., M$.

The BSC channels are assumed to be memoryless and independent, hence the sequence X_i is also an $i.i.d$ sequence. For equal observation accuracy in sensors, $\beta_i = \beta$, the pairwise correlation parameter between each set of two sources can be modeled as follows [7]

$$\begin{aligned} &p(x_i^k = \alpha | x_j^k = \alpha) = 1 + 2\beta_i\beta_j - \beta_i - \beta_j = 1 + 2\beta^2 - 2\beta \\ &p(x_i^k = \alpha | x_j^k = \bar{\alpha}) = \beta_i + \beta_j - 2\beta_i\beta_j = 2\beta - 2\beta^2 \\ &\alpha \in \{0, 1\} \quad , \quad \bar{\alpha} = 1 - \alpha \end{aligned} \tag{2}$$

This means that data correlation between the i^{th} and j^{th} sensors can be modeled as a BSC channel with crossover probability $\beta_{ij} = 2\beta - 2\beta^2$. This result is used in decoding algorithm as follows in section II-C.

B. Encoder Design

The system composed of M sensors is depicted in Fig. 1. Each sensor picks a frame of length N and encodes data using an RSC encoder with generator matrix $[1\ \frac{1+D^2}{1+D+D^2}]$. The resulting systematic and parity bits form a rate $\frac{1}{2}$ encoder that can be punctured properly to achieve a desired coding rate per sensor. Random interleaving is performed prior to RSC encoding to increase the minimum distance of the resulting codeword. These sensors together form a distributed structure for PCCC. The property of correlated inputs for RSC encoders is considered in design of the decoder structure.

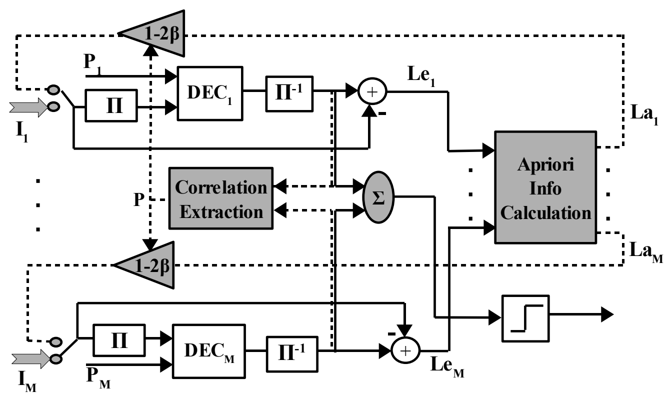

Fig. 2. Decoder structure based on multiple turbo decoder.

C. Decoder Design

Inspired by the structure of encoder, a decoder based on Multiple Turbo Decoder (MTD) can be utilized at the receiver. Log-MAP algorithm is used to calculate output Log Likelihood Ratios (LLRs). The modifications to the decoder structure are marked with dark color in Fig. 2 and summarized as follows:

- Initialization: Each RSC decoder in the decoder structure corresponds to a sensor node. At the first decoding iteration, LLRs of the systematic bits are fed to all RSC decoders rather than only a single one. This results in biasing the final output of the decoder to the source data rather than biasing to a particular sensor's observation.
- LLR exchange: The input LLR for each decoder is calculated as the average over the output LLRs of all other RSC decoders. The inter-sensor crossover probability stated in section II-A is taken into account in this process. The extrinsic LLRs of each decoder is scaled down according to $L_{a_i} = (1-2\hat{\beta})\frac{1}{M-1}\sum_{j=1,j\neq i}^{M} L_{e_j}$ before applying as apriori information to the other decoders, where L_{a_i} and L_{e_i} are apriori and extrinsic LLRs of the i^{th} RSC decoder and $\hat{\beta}$ is the estimation of β.
- Decision: In the last iteration, hard decision is made based on the average of output LLRs, $L_{av}^{out} = \frac{1}{M}\sum_{i=1}^{M} L_i^{out}$ instead of the output LLR of a particular RSC decoder, for the same reason as initialization step.

Since the BSC crossover probability between source data and each sensor is required in the decoding algorithm, it is estimated using the output LLRs after each iteration. The data observation corresponding to each sensor is estimated based on the sign of LLR of RSC decoders that gets closer to real observation of the sensor after each iteration. If $Y_i = \{y_i^1, y_i^2, ..., y_i^N\}$ denotes the transmitted BPSK modulated frame by the i^{th} sensor and $\hat{Y}_i$ shows its estimation at the end of each iteration we have,

$$\hat{y}_i^k \approx y_i^k = 2x_i^k - 1 = 2(s^k \oplus e_i^k) - 1 \tag{3}$$

A new random variable ρ_{ij}^k is defined as

$$\rho_{ij}^k = \frac{|\ \hat{y}_i^k - \hat{y}_j^k\ |}{2} \tag{4}$$

It can be shown that the average of ρ_{ij} over received frame defined as $\hat{\beta}_{ij} = \frac{\sum_{k=1}^{N}\rho_{ij}^k}{N}$ is a random variable with mean $\beta_{ij} = 2\beta - 2\beta^2$ and variance $\frac{1}{N}(\beta_{ij} - \beta_{ij}^2)$. The variance approaches zero for large frame length N.

For more accuracy, inter-sensor crossover probability is estimated by calculating the average of the $M-1$ consecutive inter-sensor correlation as $\frac{\sum_{i=1}^{M-1}\hat{\beta}_{i,i+1}}{(M-1)}$. The simulation results verify the accuracy of this method.

III. Distributed Space Time Coding in Sink Nodes

In the proposed model in Fig. 1, instead of commonly used single sink configuration, two sinks operating at amplify and forward mode are placed at each cluster. The sink nodes operate in parallel at the same frequency band. A distributed version of 2×1 STBC is utilized at sink nodes to improve the BER performance exploiting diversity gain.

In most recent works, one or more additional relay nodes are added to the system in order to implement cooperative diversity [8], while in our proposed scenario, both sink nodes have a noisy version of the received frames, hence communication between sink nodes is not required anymore.

In order to achieve optimum performance for a given total power consumption, it has been shown that the power should be equally divided between transmitters and relay nodes [9]. Therefore, if the total power is P_t, powers $\frac{P_t}{2M}$ and $\frac{P_t}{4}$ are assigned to the sensors and to each of the sink nodes, respectively.

IV. Performance Analysis

An upper-bound for the system BER performance is derived in this section. The transmission links from each sensor to the base station, point C to D in Fig. 1, form a distributed space-time codes. The following bit error probability equation for this link is derived as a function of SNR in [9].

$$P_b^{in} = \frac{4}{\pi}\int_0^{\pi/2}\left[\int_0^{\infty}\frac{e^{-\frac{2x}{\sin^2\phi}}}{\mu}K_0\Big(\sqrt{\frac{4x}{\mu}}\Big)dx\right]^2 d\phi \tag{5}$$

where P_b^{in} is probability of bit error, K_0 is modified zero order Bessel function of second kind and μ is the equivalent SNR of the system.

In order to derive an upper bound for system BER performance, we consider a *basic decoder* with independent MAP decoding for each RSC endoder followed by a ML decoder. The channel from each sensor to the base station, point C to D in Fig. 1 is replaced with a BSC channel with crossover probability P_b^{in}.

The bit Weight Enumeration Function (WEF) for the utilized RSC encoders is calculated following the procedure in [10] as

$$\begin{aligned} B(X) &= \frac{3X^5 - 6X^6 + 2X^7}{1 - 4X + 4X^2} \\ &= 3X^5 + 6X^6 + 14X^7 + 32X^8 + \ldots \end{aligned} \tag{6}$$

An upper-bound for the bit error probability of a RSC encoder over a BSC channel with small crossover probability P_b^{in}, according to [10] is

$$P_b^{rsc} < B(X)|_{X=2\sqrt{P_b^{in}\bar{P}_b^{in}}} \approx B_{dfree}[2\sqrt{P_b^{in}\bar{P}_b^{in}}]^{d_{free}} \tag{7}$$

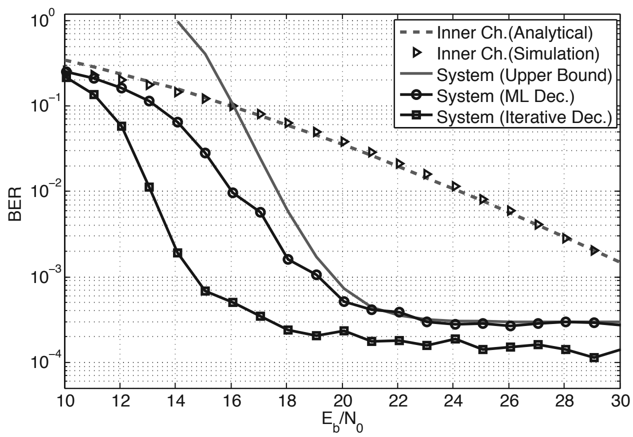

Fig. 3. BER performance comparison for the proposed decoding scheme with standard decoder, number of sensors: 4, frame length: 256, Observation accuracy is modeled as a BSC channel with crossover probability: 0.01.

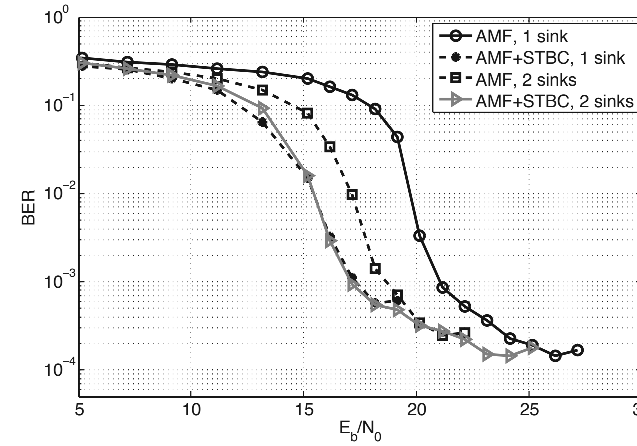

Fig. 4. BER performance of the proposed coding scheme with one and two sinks operating in amplify and forward mode with and without DSTBC.

where $d_{free} = 5$ is the minimum distance of the code and B_{dfree} is 3. Hence, the probability of bit error for each observation, after RSC decoding with respect to the source signal, point A in Fig. 1, considering observation inaccuracy parameter β_i is upper bounded by

$$P_b^i < \beta_i \otimes P_b^{rsc} = \beta_i(1 - P_b^{rsc}) + (1 - \beta_i)P_b^{rsc} \quad (8)$$

In the output of *basic decoder*, a particular bit is in error, if the corresponding bit in more than half of the sensors decoded incorrectly. Hence, the final bit error probability of the system, from point A to E in Fig. 1, is upper-bounded as

$$P_b^s < \sum_{i=\lfloor \frac{M}{2} \rfloor+1}^{M} \binom{M}{i} p^i q^{M-i} + \frac{(-1)^M + 1}{4} \binom{M}{\frac{M}{2}} (pq)^{\frac{M}{2}}$$
$$p = 1 - q = P_b^i \quad (9)$$

V. Simulation Results

The system BER performance versus $E_b/N_0 = M\frac{SNR}{R}$ is presented in Fig. 3, where E_b is total energy per information bit, N_0 is one sided noise power, M is the number of sensors and R is the coding rate at each sensor.

Dashed line in Fig. 3 shows the analytical probability of bit error based on equation (5) for inner link from the sensor outputs to base station, point C to E in Fig. 1, which matches the simulation results. Solid line in Fig. 3 shows the upper bound of the overall system BER performance derived in equation (9). Simulation results show that the performance of *basic decoder* approaches the upper bound for high SNR values. It can be seen that the proposed iterative decoder outperforms the basic decoder. The performance improvement ranges from 1 to 4 dB for different SNR values. However, the error floor of the overall system is not improved considerably since it is imposed by the number of sensors.

Fig. 4 presents the performance improvement gained by the proposed model employing two sink nodes and utilizing DSTBC in a two-tiered network. It is shown that using two sinks increases the performance by about 2 to 3 dB due to the diversity gain. Also 1 to 2 dB more additional improvement is obtained using DSTBC with the same power consumption.

VI. Conclusion

In this article, a new DJSCC scheme with iterative decoding is proposed for a two-tiered WSN. The proposed scheme is easily expandable to a large number of sensors, appropriate for applications with indirect source observation. With the self correlation estimation property of the proposed algorithm, it is applicable to unknow and even time-varying observation accuracy models. An analytical upper bound on BER performance is derived for the system model and verified with the simulation results. The BER performance of the system with the proposed algorithm shows 1 to 4 dB improvement compared to the basic decoding scheme.

The proposed use of secondary sink node at each cluster employing DSTBC not only increases the reliability of the system in case of failure in one of sink nodes, but also increases the BER performance of the whole system by about 2 to 3 dB.

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